MODELING AND SIMULATION OF A STEWART MECHANISM

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Abstract: This paper presents a full analysis of the dynamic behavior of a six- degree-of-freedom Stewart mechanism. In existing dynamic models of this mechanism, derived using either the Lagrange formulation or the Newton-Euler method, the masses of the supporting legs often have been ignored and, their inertia, invariably, assumed negligible. The modeling scheme developed in this paper is based on Kane's dynamical equations and takes account of both the platform's and the supporting legs' masses and inertia. A computer program is reported which implements this scheme to determine the actuator forces needed to produce desired platform trajectories. Finally, the outputs of this program are presented, first to highlight the effects of supporting leg dynamics, and second, to show the even greater influence of drive system inertia on the overall dynamics of the mechanism.

Keywords: Stewart mechanism; dynamics modeling; Kane's equations; aircraft simulator; robot manipulator.

1. Introduction

From the point of view of kinematics, robot manipulators can broadly be separated into two categories, namely, serial manipulators and parallel manipulators. Serial manipulators are difficult to design for applications involving fine positioning, large payloads or fast response times. In contrast, parallel manipulators have a very fine positioning control ability due to their high power-to-weight and stiffness-to-weight ratios.

Most parallel manipulators developed to-date are variations of the Stewart aircraft simulator [1], which is a triangulated structure comprising a mobile platform supported by six extendible legs. A literature survey has indicated that several methods [2-5] based on either the Lagrangian formulation or the Newton-Euler method have been proposed for the dynamic analysis of that device. However, leg masses often have been ignored and inertia, invariably assumed negligible. An alternative to the Lagrangian and the Newton-Euler techniques is Kane's formulation [6,7]. Kane's method has been adopted for modeling work reported in this paper. One of the main aims of this work was to develop a full dynamic model of the Stewart mechanism taking into account both the masses and inertia of the platform and supporting legs.

The paper comprises three main sections. Section 2 presents the dynamics modeling of the Stewart mechanism using Kane's method. Section 3 gives the results of a simulation program implementing the dynamics model, thus revealing the effects of leg masses, leg inertia and drive system inertia on the dynamic behavior of the platform. Finally, in Section 4, a global space trajectory controller for a computer-driven six-degree-of-freedom micro-manipulator based on the Stewart mechanism design is reported together with experimental results obtained from the controller and the manipulator.

2. Modeling

The six-degree-of-freedom Stewart mechanism configuration chosen for this study is depicted in Fig. 1. The mechanism consists of a movable platform, a base, and six supporting legs L_i (i=1,...,6),which link the platform to the base through ball joints at the both ends of each leg. A leg consists of two parts, the upper leg B_i and the lower leg A_i. The upper leg B_i undergoes pure translation relative to A_i. . Coordinates $(x, y, z, \theta_x, \theta_y, \theta_z)$ are assigned to the platform, defining its position and orientation with respect to an inertial coordinate frame N fixed to the base. Likewise six "generalized" coordinates are assigned to each of the lower and upper supporting legs. The overall lengths l_j (j=1,...,6) of the legs, which are independently controlled, are taken as the "generalized independent" coordinates. (See Appendix A for an outline of Kane's dynamics modeling approach and the terminology employed). The angular and linear velocities of the platform as linear combinations of generalized independent speeds are given by:-

$$\underline{\omega}_p = \sum_{j=1}^{6} \underline{\tilde{\omega}}_{jp} \dot{l}_j \tag{1}$$

and

$$V_p = \sum_{j=1}^{6} \underline{\tilde{V}}_{jp} \dot{l}_j \tag{2}$$

The vector terms $\underline{\tilde{\omega}}_{jp}$ and $\underline{\tilde{V}}_{jp}$ in Eq. (1) and Eq. (2) are, respectively, the non-holonomic partial angular velocity and non-holonomic partial linear velocity of the platform, associated with generalized independent speeds $\dot{l}_j$.

The lower supporting leg A_i and upper supporting leg B_i have the same angular velocity :-

$$\underline{\omega}_{L_i} = \sum \underline{\tilde{\omega}}_{jL_i} \dot{l}_j$$

The linear velocities of A_i and B_i are :-

$$\underline{V}_{A_i} = \sum_{j=1}^{6} \underline{V}_{jA_i} \dot{l}_j$$

and

$$\underline{V}_{B_i} = \sum_{j=1}^{6} \underline{V}_{jB_i} \dot{l}_j$$

Again, $\underline{\tilde{\omega}}_{jL_i}$, $\underline{\tilde{V}}_{jA_i}$ and $\underline{\tilde{V}}_{jB_i}$ are the non-holonomic partial velocities of the supporting legs associated with generalized independent speeds $\dot{l}_j$.

The inertia forces on the platform P, the lower supporting legs A_i and upper supporting legs B_i are, respectively,

$$\overset{*}{F} = -m_p \sum_{j=1}^{6} \left(\underline{\tilde{V}}_{jp} \ddot{l}_j \quad \underline{\dot{\tilde{V}}}_{jp} \dot{l}_j \right) \tag{3}$$

$$\overset{*}{\underline{F}}_{A_i} = -m_{A_i} \left(\sum_{j=1}^{6} \left(\underline{\tilde{V}}_{jA_i} \ddot{l}_j + \underline{\dot{\tilde{V}}}_{jA_i} \dot{l}_j \right) + \left(\underline{w}_{L_i} \times \underline{V}_{A_i} \right) \right)$$

$$\overset{*}{\underline{F}}_{B_i} = -m_{B_i} \left(\sum_{j=1}^{6} \left(\underline{\tilde{V}}_{jB_i} \ddot{l}_j + \underline{\dot{\tilde{V}}}_{jB_i} \dot{l}_j \right. + \left(\underline{w}_{L_i} \times \underline{V}_{B_i} \right) + \frac{\underline{l}_i}{l_i} \ddot{l}_i + \left(2\underline{w}_{L_i} \times \underline{l} \right) \right)$$

In the above equations m_p, m_{A_i} and m_{B_i} represent the masses of the platform, lower supporting legs and upper supporting legs.

The local inertia torques on the platform, the lower and the upper supporting legs are, respectively

$$\overset{*}{\underline{T}}_{p(loc)} = -\underline{\alpha}_{P(loc)}.I_P - \underline{\omega}_{P(loc)} \times I_P.\underline{\omega}_{P(loc)} \tag{4}$$

$$\overset{*}{\underline{T}}_{Ai(loc)} = -\underline{\alpha}_{Li(loc)}.I_{Ai} - \underline{\omega}_{Li(loc)} \times I_{Ai}.\underline{\omega}_{Li(loc)}$$

$$\overset{*}{\underline{T}}_{Bi(loc)} = -\underline{\alpha}_{Li(loc)}.I_{Bi} - \underline{\omega}_{Li(loc)} \times I_{Bi}.\underline{\omega}_{Li(loc)}$$

In Eq. (4), I_P is the inertia matrix of the platform about its principal axes. The latter are adopted as the local coordinate frame $\{C\}$ fixed to the platform's centre of mass $\underline{\omega}_{P(loc)}$ and $\underline{\alpha}_{P(loc)}$ are, respectively, the angular velocity and angular acceleration of the platform defined in $\{C\}$. Similarly, the inertia matrices I_{A_i} and I_{B_i}, the angular velocities $\underline{\omega}_{L_i(loc)}$ and angular accelerations $\underline{\alpha}_{L_i(loc)}$ are about the respective local coordinate frames $\{a_i\}$ and $\{b_i\}$ fixed to the supporting legs A_i and B_i at their centre of mass. The above local inertia torques are re-expressed in the inertial coordinate frame $\{N\}$ thus:-

$$\overset{*}{\underline{T}}_P = [R_P(t)]\overset{*}{\underline{T}}_{P(loc)} \tag{5}$$

$$\overset{*}{\underline{T}}_{Ai} = [R_{Li}(t)]\overset{*}{\underline{T}}_{Ai(loc)}$$

The matrices $[R_P(t)]$ and $[R_{L_i}(t)]$, which are functions of time t, relate the coordinate frames $\{C\}$, $\{a_i\}$ and $\{b_i\}$ and to the inertial coordinate frame $\{N\}$.

The generalized inertia forces on the platform are given by:-

$$(\overset{*}{k_j})_P = \underline{\omega}_P.\overset{*}{\underline{T}}_p + \underline{V}_P.\overset{*}{\underline{F}}_p \qquad (j = 1,...,6)$$

Rearranging the equations for $(\overset{*}{k_j})_P$ into vector form yields:-

$$\overset{*}{\underline{K}}_P = [m_P]\ddot{\underline{l}} + \underline{b}_P$$

where $[m_p]$ is the mass matrix of the platform, $\ddot{\underline{l}}$ the vector of generalized independent accelerations $\ddot{l}_j$ (j=1,...,6), and $\underline{b}_P$ the vector of velocity-dependent generalized inertia force components acting on the platform.

Similar expressions can be obtained for the lower and upper supporting legs. The generalized inertia forces for the overall system comprising the platform and the six lower legs and six upper legs are then:-

$$\overset{*}{\underline{K}} = [m_P]\ddot{\underline{l}} + \underline{b}_P + \sum_{i=1}^{6}([m_{Ai}]\ddot{\underline{l}} + \underline{b}_{Ai}) + \sum_{i=1}^{6}([m_{Bi}]\ddot{\underline{l}} + \underline{b}_{Bi}) \tag{6}$$

where $[m_{A_i}]$ and $[m_{B_i}]$ are the mass matrices of the lower and the upper supporting legs $\underline{b}_{A_i}$ and $\underline{b}_{B_i}$, are the vectors of velocity-dependent generalized inertia force components acting on the supporting legs A_i and B_i.

$$\overset{*}{\underline{K}} = [m]\underline{\dot{l}} + \underline{b} \tag{7}$$

where $[m]$ is the overall mass matrix and b the vector of velocity-dependent generalized inertia force component. $[m]$ is a 6 x 6 matrix. $\underline{b}$ is a 6 x 1 vector, in which those terms involving squares of velocities are centrifugal forces and those containing products of different velocities are coriolis forces.

The contribution of the "distance" forces (such as those due to gravity) to the generalized active force K_j is :-

$$(K_j)_d = \underline{\tilde{V}}_{jP} \cdot \underline{G}_P + \sum_{i=1}^{6}\left(\underline{\tilde{V}}_{jA_i} \cdot \underline{G}_{A_i} + \underline{\tilde{V}}_{jB_i} \cdot \underline{G}_{B_i}\right) \quad (i=1, \ .6) \tag{8}$$

where $\underline{G}_P$, $\underline{G}_{Ai}$ and $\underline{G}_{Bi}$ are the distance forces due to gravity on the platform P and the lower and upper supporting legs A_i and B_i.

The contribution of the contact forces to the generalized active force K_j is:-

$$(K_j)_c = \sum_{i=1}^{6}\sigma_i\left(\underline{\tilde{V}}_{jB_i} - \underline{\tilde{V}}_{jA_i}\right)\cdot\frac{\underline{l}_i}{l_i} - \sum_{i=1}^{6}u_i\left(\underline{\tilde{V}}_{jB_i} - \underline{V}_{jA_i}\right)\cdot\frac{\underline{\dot{l}}_i}{\dot{l}_i} \tag{9}$$

where σ_i and μ_i are respectively the magnitudes of actuator and friction forces exerted by A_i on B_i along $\underline{l}_i$.

The generalized active force K_j is the sum of $(K_j)_d$ and $(K_j)_c$,

$$K_j = (K_j)_d + (K_j)_c \quad (j=1, \ .6)$$

The elements K_j can be grouped into a vector $\underline{K}$ which can be expressed as:-

$$\underline{K} = \underline{g} + [a]\underline{\sigma} + \underline{f}$$

where $\underline{g}$, $[a]$, $\underline{f}$ are contributions by gravitational, actuator and friction forces, respectively.

Kane's dynamical equations are obtained by substituting Eq. (7) and Eq. (10), such that:-

$$\overset{*}{\underline{K}} + \underline{K} = 0$$

which yields:-

$$[m]\underline{\dot{l}} + \underline{b} + \underline{g} + [a]\underline{\sigma} + \underline{f} = 0$$

3. Inverse Dynamics

A Computer program has been developed for inverse dynamics simulation using Equation (11). To implement the dynamic model described in Section 3, a simulation program in Borland C has been developed for execution on a 486 DX computer or atleast a 386DX with a 387 coprocessor. This program can be applied to a variety of configurations of the Stewart mechanism to compute the actuator forces for the desired trajectory of the platform. The program computes the actuator forces $\underline{\sigma}$ when the trajectory of the platform is specified (i.e. $\underline{\ddot{l}}$ is given).

This section presents simulation results for the stewart mechanism. Different starting configurations

This section is divided into four parts. In the first part simulation results are shown without taking account of the supporting leg dynamics. In the second part, results are presented considering the platform and supporting leg dynamics. The third part presents the simulation results for the combined effects of the platform, supporting legs and the drive system dynamics. The fourth part examines and compares the results obtained in parts 3.1, 3.2 and 3.3.

3.1 Ignoring supporting leg dynamics

The simulation results of the computer program, ignoring supporting leg masses, are shown in Fig. 2. For the programs the trajectories were defined in terms of the global (platform) accelerations ($\ddot{x}$, $\ddot{y}$, $\ddot{z}$, $\ddot{\theta}_x$, $\ddot{\theta}_y$, $\ddot{\theta}_z$) and the simulations were performed over a period of 0.1 second to obtain the magnitudes of the required actuator forces σ_i (i= 1,...,6) along the longitudinal axes of supporting legs L_i The maximum values adopted for ($\ddot{x}$, $\ddot{y}$, $\ddot{z}$, $\ddot{\theta}_x$, $\ddot{\theta}_y$, $\ddot{\theta}_z$) were :- $\ddot{x} = \ddot{y} = \ddot{z} = 10\ m/s^2$, $\ddot{\theta}_x = \ddot{\theta}_y = \ddot{\theta}_z = 8\ rad/s^2$.

3.2 Effects of supporting leg dynamics

Given the same desired trajectories as in the previous section, the required actuator forces were computed using the computer program. The lower and upper supporting legs had the same mass (0.065 Kg).The simulation results for six different trajectories are shown in Fig. 3.

The effects of leg dynamics are evident by comparing the results of this part for the case where the supporting leg dynamics is taken into account with those given in previous part shows a significant increase in actuator forces for the same desired trajectories. The effect of supporting leg dynamics is uniform for all actuating forces during translation along $\underline{n}_3$. For instance, comparing the graphs of Figs. 2(c) and 3(c) shows an increase in the six required actuator forces with increasing leg masses. The results for other trajectories show the non-uniform effects of the leg dynamics. These effects are dependent on the trajectories of the platform and its starting position. For instance, comparing Fig. 3(a) with Fig. 2(a) reveals that the magnitudes of actuator forces σ_1 , σ_3 , σ_4 and σ_6 are increased whereas the magnitudes of σ_2 and σ_5 are decreased with the addition of leg masses. This is because the actuators in legs L_1 and L_5 which undergo contraction, are assisted by the weights of the legs during this movement.

3.3 Effects of drive system inertia

In this part simulation results are presented for the Stewart mechanism taking account of the platform, supporting leg and the drive system inertia. The drive system for each actuator was considered with a servomotor, a reduction gearbox and rack and pinion final drive.

For combined platform, the supporting legs and drive systems, modifying the Eq. 11 in the form:-

$$\underline{\sigma} = -[a]^{-1}(([m]+[m_d])\ddot{\underline{l}} + \underline{b} + \underline{b}' + g + f)$$

is the vector of forces transmitted from the actuators through the transmission mechanism to the mechanism. The matrix contains inertia terms for the six drive systems and vector

$$m_{ij} = 0 \quad i \neq j \quad (i, j = 1,\ldots,6) \tag{13}$$

and

$$m_{ii} = \zeta^2\ I/r^2 \tag{14}$$

ζ is the gearbox reduction ratio, I is the rotor inertia and r is the radius of the pinion.

To observe the effects of the drive system dynamics on the overall dynamic behavior of the mechanism, Eq. 12 was used in the computer program. Actuator force profiles were obtained by computer simulation for various platform trajectories. To simplify the computation, damping and friction were ignored. Figs. 4(a) and 5(a) show the actuator forces required for two different desired platform trajectories, without considering the drive system inertia. The varying actuator forces result from the dynamic coupling between the platform and the six supporting legs. Figs. 4(b) and 5(b) were computed for the respective trajectories of Figs. 4(a) and 5(a), with the drive system inertia present. From Figs. 4(b) and 5(b), it can be seen that the constant drive system inertia forces are far larger than the dynamic coupling effects on the manipulator, with actuator forces considerably large in the latter case

4. Conclusion

This paper has been about building the dynamic model of a six-degree-of-freedom Stewart mechanism and applying model for investigating the dynamic behavior and controlling the mechanism. Based on Kane's dynamical equations the model has been developed using the concept of equivalent torques about local coordinate frames. For implementing the dynamic model a computer program has been developed to compute the actuator forces for the desired trajectory of the platform. The outputs obtained form this program for the analysis of the mechanism have been presented. From these outputs it has been seen that the lower leg masses have negligible effects on the overall dynamics of the mechanism whereas the effects of upper leg masses are very significant. From the outputs obtained considering the drive system inertia it has been investigated that the dynamic coupling effects of the platform and the supporting legs are very small compared to the inertia forces of the drive systems. A global space trajectory controller for the mechanism has been presented and each of six drive systems has been treated separately as if it were moving a constant load.

5. References

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Appendix A

Kane's Dynamics Equations for Spatial Mechanisms

To describe uniquely the dynamical configurations of an f-degree-of-freedom mechanism the generalized coordinates are defined. For a mechanism with N links there can be 6N generalized coordinates q_r $(r = 1,...,6N)$ which may be measured in local coordinate frames. For a non-holonomic system, where inter-connecting links form closed loops and impose motion constraints upon one another, not all the 6N generalized coordinates are independent. A set of independent coordinates q_j $(j = 1,...,f)$ is chosen from the generalized coordinates. Geometrical constraint equations are considered to obtain the remaining coordinates q_k $(k = f+1,...,6N)$ in terms of the generalized independent coordinates q_j.

The generalized speeds u_r ($r =$.,...,$6N$) for the mechanism are defined as:-

$$u_r = q_r$$

There is also a set of generalized independent speeds u_j ($j = 1,...,f$). Kinematic constraint equations are formulated which express the remaining generalized speeds in terms of the generalized independent speeds in the form:-

$$u_k = \sum_{j=}^{f} A_{Kj} u_j + B$$

A_{kj} and B_k are functions of $q_1,...,q_f$ and time. Using the above kinematic constraint equations, the angular and linear velocities of all links are computed as linear combinations of the generalized independent speeds, namely,

$$\underline{\omega}_i = \sum_{j=1}^{f} \underline{\tilde{\omega}}_{ij} u_j$$

and

$$\underline{V}_i = \sum_{j=1}^{f} \underline{\tilde{V}}_{ij} u_j$$

The quantities $\underline{\tilde{\omega}}_{ij}$ and $\underline{\tilde{V}}_{ij}$ are known as the partial angular velocity and partial linear velocity of link L_i associated with generalized independent speed u_j $\underline{\tilde{\omega}}_{ij}$ and $\underline{\tilde{V}}_{ij}$ are functions of the generalized coordinates q_r.

The angular accelerations $\underline{\alpha}_i$ and linear accelerations $\underline{a}_i$ of all links are also expressed as linear combinations of the generalized independent accelerations which are the time derivatives of u_j.

To formulate Kane's dynamical equations, the generalized inertia forces and generalized active forces are needed. The generalized inertia forces are expressed as:-

$$\overset{*}{K}_j = \sum_{i-1}^{N} (\overset{*}{K}_j)_i \qquad (j=1,2.....f)$$

where

$$(\overset{*}{K}_j)_i = \underline{\tilde{\omega}}_{ij} \cdot \underline{\overset{*}{T}}_i + \underline{\tilde{V}}_{ij} \cdot \underline{\overset{*}{F}}_i$$

$\underline{\overset{*}{F}}_i$ and $\underline{\overset{*}{T}}_i$ are, respectively, the inertia force and inertia torque acting on the link L_i

$$\underline{\overset{*}{F}}_i = -m_i \underline{\overset{*}{a}}_i$$

and

$$\underline{\overset{*}{T}}_i = [R_i(t)]^T \underline{\overset{*}{T}}_{i(loc)}$$

where the inertia torque $\overset{*}{\underline{T}}_{i(loc)}$ is computed as:-

$$\overset{*}{\underline{T}}_{i\,(loc)} = -\underline{\alpha}_{i(loc)} \cdot I_{i(loc)} \qquad \underline{\omega}_{i(loc)} \times I_{i(loc)} \cdot \underline{\omega}_{i(loc)} \qquad \text{(A.3)}$$

In Eq. (A.3), $\overset{*}{\underline{T}}_{i(loc)}$, $\underline{\alpha}_{i(loc)}$, $\underline{\omega}_{i(loc)}$ and $I_{i(loc)}$ are the inertia torque, angular acceleration, angular velocity, and inertia matrix defined in the local coordinate frame $\{X_i\}$ of link L_i. $[R_i(t)]$ is a matrix expressing the relationship between $\{X_i\}$and $\{N\}$, the inertial coordinate frame fixed to the base of the mechanism, in which $\overset{*}{\underline{T}}_i$, $\underline{\alpha}_i$, and $\underline{\omega}_i$ are globally defined.

To formulate the expressions for the generalized active forces of the system, the active forces acting on all individual links are required. The active force applied at a point on a link is made up of two types of forces, namely, the contact forces (for example, actuator forces and friction) and the distance forces (for example, gravitational forces and magnetic forces). The generalized active forces are given as:-

$$K_j = \sum_{i=1}^{N} (K_j)_i \qquad (j = \qquad .n)$$

The active force on link L_i equivalent to a torque $\underline{T}_i$ together with a force $\underline{F}_i$ applied at a point on the link. Therefore, the generalized active force acting on Li is:-

$$\left(K_j\right) = \underline{\tilde{\omega}}_{ij} \cdot \underline{T}_i + \underline{V}_{ij} \cdot \underline{F}_i$$

Some forces that make up the active force do not contribute to the generalized active forces. In Eq. (A.5) these non-contributing forces are easily eliminated.

Kane's dynamical equations involve Eq. (A.1) and Eq. (A.4) and are written in the form:-

$$\overset{*}{K}_j + K_j = 0$$

These equations can readily be solved for the accelerations resulting from given applied forces and torques (the direct dynamics or motion simulation problem) or the forces or torques required to cause desired output accelerations (the inverse dynamics problem).